OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 2637

Mechanics 1

Thursday **14 JUNE 2001** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use $9.8\,\mathrm{m\,s^{-2}}$.
- You are permitted to use a graphic calculator in this paper.

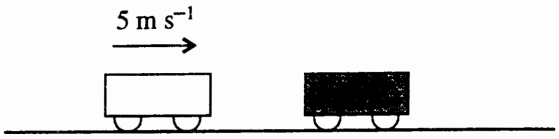
INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 4 printed pages.

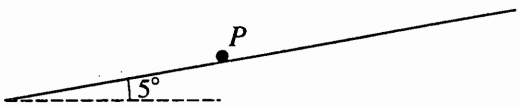
[Turn over

1



Two wagons, each of unloaded mass 1000 kg, are free to travel on a straight horizontal track. One of the wagons carries a load of mass m kg and the other is empty. The empty wagon is travelling at 5 m s^{-1} when it runs into the loaded wagon which is stationary (see diagram). Immediately after the collision the empty wagon continues in the same direction with speed 0.5 m s^{-1}, and the loaded wagon starts to move with speed 1.5 m s^{-1}. Find the value of m. [4]

2



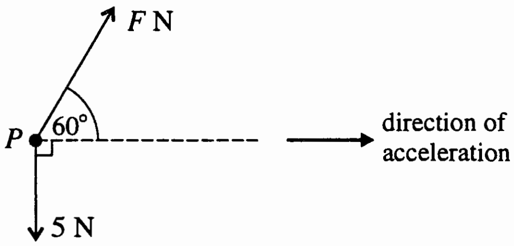
A particle P is released from rest at the top of a smooth plane, of length 5 m, which is inclined at 5° to the horizontal (see diagram). Air resistance may be neglected. Find

(i) the acceleration of P down the plane, [2]

(ii) the time taken for P to reach the bottom of the plane, [2]

(iii) the speed with which P reaches the bottom of the plane. [2]

3



A particle P of mass 0.4 kg can move on a smooth horizontal table. P is acted on by two horizontal forces of magnitudes F N and 5 N. The direction of the acceleration of P makes angles of 60° and 90° with the directions of the forces of magnitudes F N and 5 N respectively, as shown in the diagram.

(i) State the direction of the resultant of the two forces. [1]

(ii) Find the value of F. [2]

(iii) Find the magnitude of the resultant of the two forces, and find also the magnitude of the acceleration of P. [4]

4

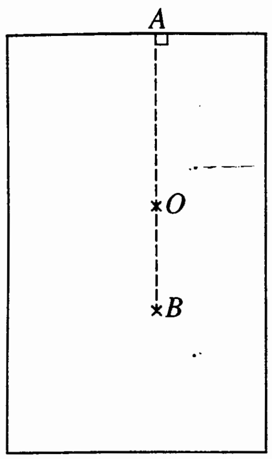


Fig. 1

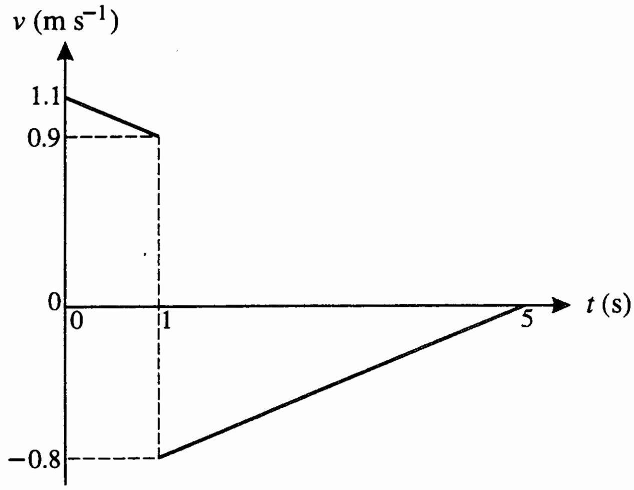


Fig. 2

A ball is at the point O on a snooker table when it is set in motion along the length of the table towards the point A at the end of the table. When the ball reaches A it rebounds along the line AO and comes to rest at B (see Fig. 1). The (t, v) graph for the motion is shown in Fig. 2.

(i) Find the distance OA. [2]

(ii) Find the distance OB. [3]

(iii) Show that the deceleration of the ball from O to A is the same as that from A to B. [3]

5 A train of mass 200 000 kg makes a journey, on a straight horizontal track, consisting of the following three stages.

First stage:	The train moves with constant acceleration, starting from rest and reaching a speed of 25 m s^{-1} when it has travelled 0.25 km.
Second stage:	The train travels 11.25 km at a constant speed of 25 m s^{-1}.
Third stage:	The train has constant deceleration and comes to rest in 40 s.

(i) Find the acceleration during the first stage. [3]

(ii) Find the total distance travelled. [2]

(iii) The resistance to motion is modelled as a constant force of magnitude 100 000 N throughout each stage of the journey. Find the driving force produced by the train's engine during the first stage. [2]

(iv) During the third stage there is no driving force produced by the train's engine, but a retarding force is produced by the train's brakes. Find this retarding force. [4]

[Turn over

6 A particle starts from rest at O and travels in a straight line to A. The time taken for the journey from O to A is 25 s, and the particle reaches A with a speed of $15\,\mathrm{m\,s^{-1}}$. The velocity of the particle t seconds after it leaves O is $v\,\mathrm{m\,s^{-1}}$. It is given that $v = kt^2$, where k is a constant.

(i) Show that $k = \frac{3}{125}$. [2]

(ii) Find the distance OA. [4]

(iii) Find the distance of the particle from O when its acceleration is $0.72\,\mathrm{m\,s^{-2}}$. [6]

7

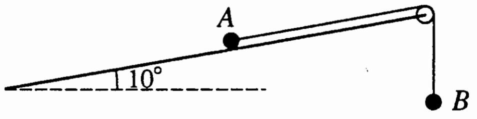


Fig. 1

Two particles A and B, of masses 0.5 kg and m kg respectively, are joined by a light inextensible string. Particle A rests on a board which is fixed at 10° to the horizontal. The coefficient of friction between A and the board is 0.15. The string is parallel to a line of greatest slope of the board and passes over a small smooth pulley at the top of the board. Particle B hangs vertically below the pulley, as shown in Fig. 1. The system is in equilibrium.

(i) (a) Given that A is on the point of sliding up the plane, show that $m = 0.161$, correct to 3 decimal places.

(b) Given instead that A is on the point of sliding down the plane, show that $m = 0.013$, correct to 3 decimal places.

[7]

The board is now fixed horizontally and A is held on the board. The string passes over the pulley and B hangs vertically below the pulley as shown in Fig. 2. A is now released.

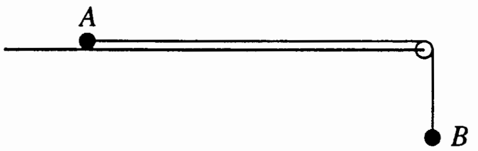


Fig. 2

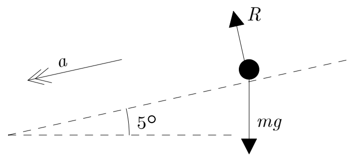
(ii) Show that the system starts to move if m has the value in part **(i) (a)**, but remains at rest if m has the value in part **(i) (b)**. [3]

(iii) Find the magnitude of the frictional force acting on A for each of the values of m in part **(i)**. [2]

1 conservation of momentum ….

$$1000\times 5 = 1000\times 0\cdot 5 \ + \ (1000+m)\times 1\cdot 5 \qquad 1000+m = 3000 \qquad m = \mathbf{2000}$$ [4]

2



N2 (down slope)

$$mg\sin 5^\circ = ma$$

$$a = 9\cdot 8\sin 5^\circ = 0\cdot 854126... = \mathbf{0\cdot 854} \quad \text{(3 s.f.)}$$

[2]

time to reach bottom …..

$$s = ut + \tfrac{1}{2}at^2$$

$$5 = \tfrac{1}{2}\times 0\cdot 854126\times t^2 \qquad t = 3\cdot 42167... = \mathbf{3\cdot 42\ s} \ \text{(3 s.f.)}$$

[2]

speed at bottom …

$$v^2 = u^2 + 2as \qquad v^2 = 8\cdot 54126.... \qquad v = 2\cdot 92254... = \mathbf{2\cdot 92\ ms^{-1}}$$

[2]

3 the resultant is in the direction of the acceleration.

[1]

normal to the direction of acceleration …

$$\text{N2} \qquad F\sin 60^\circ - 5 = 0 \qquad F = {}^{5}\!\!\diagup_{\sin 60^\circ} = 5\cdot 7735... = \mathbf{5\cdot 77} \quad \text{(3 s.f.)}$$

[2]

$$\text{magnitude of resultant} = 5\cos 60^\circ = \mathbf{2\cdot 5\ N} \qquad \text{acceleration} = {}^{2\cdot 5}\!\!\diagup_{0\cdot 4} = \mathbf{6\cdot 25\ ms^{-2}}$$

[4]

4

$$OA = \text{area of trapezium} = 1\times\left(\frac{1\cdot 1 + 0\cdot 9}{2}\right) = \mathbf{1\ m}$$

[2]

$$AB = \text{area of triangle} = \tfrac{1}{2}\times 4\times 0\cdot 8 = 1\cdot 6\,\text{m} \qquad \therefore OB = 1\cdot 6 - 1 = \mathbf{0\cdot 6\ m}$$

[3]

$$\text{deceleration}_{OA} = {}^{0\cdot 2}\!\!\diagup_{1} = 0\cdot 2\ \text{ms}^{-2} \qquad \text{deceleration}_{AB} = {}^{0\cdot 8}\!\!\diagup_{4} = 0\cdot 2\ \text{ms}^{-2}$$ **the same!**

[3]

5 first stage …

$v^2 = u^2 + 2as$ $\qquad 25^2 = 0^2 + 2a \times 250$ $\qquad a = \mathbf{1 \cdot 25\ ms^{-2}}$

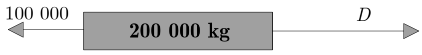
[3]

third stage …

$$s = \left(\frac{u+v}{2}\right)t = 40\left(\frac{25+0}{2}\right) = 600\ \text{m}$$

total distance travelled $= 0 \cdot 25 + 11 \cdot 25 + 0 \cdot 6 = \mathbf{12 \cdot 1\ km}$

[2]



N2 $(\rightarrow)$

$D - 100\,000 = 200\,000 \times 1 \cdot 25$

$D = \mathbf{350\,000\ N}$

[2]

third stage …

$a = {}^{-25}\!/_{40} = {}^{-}0 \cdot 625$ $\qquad {}^{-}R - 100\,000 = 200\,000 \times {}^{-}0 \cdot 625$ $\qquad R = \mathbf{25\,000\ N}$

[4]

6 $v = k\,t^2$ and $v = 15$ when $t = 25$ $\quad \Rightarrow \quad 15 = 625k \quad \Rightarrow \quad k = {}^{15}\!/_{625} = \mathbf{{}^{3}\!/_{125}}$ (show)

[2]

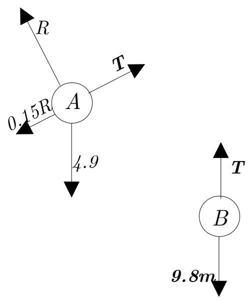
$x = \int v\,\mathrm{d}t = \int \tfrac{3}{125}t^2\,\mathrm{d}t = \tfrac{1}{125}t^3$ $\qquad \therefore OA = \tfrac{1}{125} \times 25^3 = \mathbf{125\ m}$

[4]

$a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = \tfrac{6}{125}t$ $\qquad$ so when $a = 0 \cdot 72,\quad t = 15\,\text{s}$ $\qquad \therefore x = \tfrac{1}{125} \times 15^3 = \mathbf{27\ m}$

[6]

7



on the point of sliding **up** the plane …

$R = 4 \cdot 9\cos 10^\circ = 4 \cdot 82555...$

$$\left.\begin{array}{ll} \text{N2}(A) & T - 0 \cdot 15R - 4 \cdot 9\sin 10^\circ = 0 \\ \text{N2}(B) & 9 \cdot 8m - T = 0 \end{array}\right\}$$

$9 \cdot 8m = 0 \cdot 15 \times 4 \cdot 82555... + 4 \cdot 9\sin 10^\circ$

$m = 0 \cdot 160684... = \mathbf{0 \cdot 161}$ (3 d.p.) (show)

when on the point of sliding **down** the plane …

$9 \cdot 8m = 4 \cdot 9\sin 10^\circ - 0 \cdot 15 \times 4 \cdot 82555...$ $\qquad m = 0 \cdot 012963... = \mathbf{0 \cdot 013}$ (3 d.p.)

[7]

horizontal board …

limiting friction on A $= 0 \cdot 15 \times 0 \cdot 5 \times 9 \cdot 8 = 0 \cdot 735$

[2]

weight of B is …..

Total [60]